

Dundigal, Medchal Dist. Hyderabad - 500043, Telangana.

# I B.Tech I Sem Regular Examination, December 2019 **MATHEMATICS-1** (CIVIL, EEE, MECH, ECE, CSE & IT)

# Time: 3 Hours.

Note: 1. This question paper contains two parts A and B.

- 2. Part- A is Compulsory. Answer all Questions which carries 20 marks.
- 3. Part B consists 5 units. Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

# **PART-A**

# (10\*2 Marks=20Marks)

1.	a)	Find the values of b such that the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3	2M
	b)	Define Echelon form of a matrix with a suitable example.	2M
	c)	Determine $A^8$ for the matrix $= \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ .	2M
	d)	Discuss about nature of the quadratic form.	2M
	e)	Test for the convergence of the series $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$ .	2M
	f)	Define Absolute and conditional convergent.	2M
	g)	Find the value of $\Gamma\left(-\frac{1}{2}\right)$ .	2M
	h)	State Rolle's theorem	2M
	i)	If $u = x^2 - y^2$ , $x = 2r - 3s + 4$ , $y = -r + 8s - 5$ . Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial x}$ .	2M

1) If 
$$u = x^2 - y^2$$
,  $x = 2r - 3s + 4$ ,  $y = -r + 8s - 5$ . Find  $\frac{1}{\partial r}$  and  $\frac{1}{\partial s}$ . 2M  
Find the stationary points of  $u(x, y) = \sin x \sin y \sin (x + y)$   
Where  $0 \le x, y \le \pi$ . 2M

Where 0 < x,  $y < \pi$ .

#### PART - B

# (5\*10 Marks=50Marks)

For what values of  $\lambda$ , the system of equations

#### x + y + z = 1, $x + 2y + 4z = \lambda$ , $x + 4y + 10z = \lambda^2$ have a solution and 10M 2 solve them completely in each case.

# OR

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Find the values of *a* and *b* for which the equations

10M x + ay + z = 3; x + 2y + 2z = b; x + 5y + 3z = 9 are consistent. When will these equations have a unique solution?

Max. Marks: 70

Reduce the following quadratic form into a sum of squares by an orthogonal transformation 10M

$$8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$$

OR

State and verify Cayley - Hamilton theorem and use it to find the inverse of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}.$ 10M

6 a) Test for convergence of the series 
$$1 - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \cdots$$
 5M

b) Test the convergence of the series  $x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \cdots$ 5M OR

7 a) Test for convergence 
$$\sum_{2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n(n+1)(n+2)}}$$
. 5M

b) Test the convergence of the series 
$$1 + \frac{x}{2} + \frac{2!x^2}{3^2} + \frac{3!x^3}{4^3} + \cdots$$
 5M

State and Verify Lagrange's mean value theorem for  

$$f(x) = \begin{cases} x \sin \frac{1}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases} \text{ in [-1, 1].}$$
10M

a) Show that  $h < e^h - 1 < he^h$  for  $h \neq 0$ . 9 5M Expand log x in powers of (x - 1) and hence evaluate log 1.1 correct to four 5M b) decimal places.

10 a) If z is a homogenous function x, y of order n then prove that 
$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz$$
. 5M

b) If 
$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$
,  $x^2 + y^2 + z^2 \neq 0$  then evaluate  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ . 5M

#### OR

Find the volume of the greatest rectangular parallelepiped that can be inscribed in the 5M

11 a) ellipsoid 
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1..$$
 5M

b) If 
$$x = u(1 - v)$$
;  $y = uv$  prove that  $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(u,v)}{\partial(u,v)} = 1.$  5M

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