



I B.TECH I Sem Supply End Examination, July 2021

**MATHEMATICS-I**  
**(CE, EEE, ME, ECE, CSE & INF)**

Time: 3 Hours.

Max. Marks: 70

Note: 1. Answer any FIVE questions.

2. Each question carries 14 marks and may have a, b as sub questions.

- Find the values of  $k$  for which the system of equations
- 1 a)  $(3k - 8)x + 3y + 3z = 0; 3x + (3k - 8)y + 3z = 0;$   
 $3x + 3y + (3k - 8)z = 0$  has a non-trivial solution. 7M CO BL
- b) Find the rank of the matrix  $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$  7M CO BL
- 2 Solve the equations  $5x + 2y + z = 12; x + 4y + 2z = 15;$   
 $x + 2y + 5z = 20$  by Gauss-Seidel method 14M CO BL
- 3 a) Show that the two matrices  $A, A^T$  have the same latent roots. 7M CO BL
- b) For a matrix  $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$  find the eigen values of  $3A^3 + 5A^2 - 6A + 2I$ . 7M CO BL
- 4 a) Show by Cauchy integral test that the series  $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$  converges if  
 $p > 1$  and diverges if  $0 < p < 1$ . 7M CO BL
- b) Discuss the convergence for  $1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$  7M CO BL
- 5 Find a modal matrix that will diagonalize the real symmetric matrix  
 $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ . Also write the resulting diagonal matrix. 14M CO BL
- 6 a) Evaluate  $\iiint xyz dx dy dz$  over the positive octant of the sphere  
 $x^2 + y^2 + z^2 = a^2$ . 7M CO BL
- b) Find the volume of solid of revolution obtained by revolving the ellipse  
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  about  $x$ -axis. 7M CO BL
- 7 a) Prove that  $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$ . 7M CO BL
- b) Verify Euler's theorem for the function  $u = \frac{x^{2/3} + y^{2/3}}{x^n + y^n}$  7M CO BL
- 8 a) Find the shortest distance from origin to the surface  $xyz^2 = 2$ . 7M CO BL
- b) If  $u = e^{a\theta} \cos(a \log r)$  then show that  $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$ . 7M CO BL