



I B.Tech I Sem Supplementary Examination, October 2022

Mathematics - I

(Common to all branches)

Time: 3 Hours.**Max. Marks: 70**

Note: 1. Question paper consists: Part-A and Part-B.

2. In Part - A, answer all questions which carries 20 marks.

3. In Part - B, answer any one question from each unit.

Each question carries 10 marks and may have a, b as sub questions.

PART- A**(10*2 Marks = 20 Marks)**

1. a) Find the rank of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & 0 & 2 \end{bmatrix}$ 2M CO1 L1
- b) Define an orthogonal matrix and give an example. 2M CO1 L1
Find the sum and product of the eigen values of the matrix
- c) $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$ 2M CO2 L2
- d) The product of two eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 2M CO2 L3
16. Find the third eigen value of A.
- e) State Cauchy's Root test. 2M CO3 L2
- f) State Cauchy's integral test. 2M CO3 L2
- g) State Rolle's theorem 2M CO4 L2
- h) Find the value of $\beta(5,4)$ 2M CO4 L4
- i) If $z = \log(x^2 + y^2)$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. 2M CO5 L2
- j) If $x = r \cos \theta, y = r \sin \theta$ then evaluate Jacobian of r, θ with respect to x, y . 2M CO5 L5

PART- B**(10*5 Marks = 50 Marks)**

- 2 a) Reduce the Matrix $A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -8 & 3 & 6 & 6 & 12 \end{bmatrix}$ into Echelon form. 5M CO1 L3
- b) Solve $x + y + z = 9, 2x + 5y + 7z = 52, 2x + y - z = 0$ by using Gauss elimination method. 5M CO1 L1

OR

- 3 a) Reduce the matrix $\begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ to Normal form. 5M CO1 L5
- b) Find whether the following equations are consistent if so solve them $2x - 3y + 7z = 5, 3x + y - 3z = 13, 2x + 19y - 47z = 32$. 5M CO1 L6

- 4 a) Find the eigen values and eigen vector of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ 5M C02 L1
- b) If $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ then find A^{-1} by using the Cayley-Hamilton theorem. 5M C02 L3

OR

- 5 a) Reduce the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. 5M C02 L2
- b) Write the matrix of the quadratic form $x^2 + 2y^2 - 3z^2$ and find the index and signature of the quadratic form. 5M C02 L1

- 6 a) Test whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+\sqrt{n+1}}}$ is convergent. 5M C03 L4
- b) Test whether the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ is convergent. 5M C03 L4

OR

- 7 a) Test for convergence the series $x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1.3}{2.4} \cdot \frac{x^5}{5} + \frac{1.3.5}{2.4.6} \cdot \frac{x^7}{7} + \dots$ 5M C03 L4
- b) Test for convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n(n+1)(n+2)}}$. 5M C03 L4

- 8 a) Verify Lagrange's theorem for $f(x) = x(x-2)(x-3)$ in the interval $[0, 4]$. 5M C04 L5
- b) Obtain Taylor's series expansion of e^x in powers of $(x-1)$ upto 5 terms. 5M C04 L6

OR

- 9 a) Verify Cauchy's mean value theorem for the functions $\sin x$ and $\cos x$ in the interval $[a, b]$. 5M C04 L5
- b) Show that $\int_0^{\pi/2} \sqrt{\cot \theta} d\theta = \frac{1}{2} \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right)$ 5M C04 L2

- 10 a) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$. 5M C05 L5
- b) If $u = x \log xy$ where $x^3 + y^3 + 3xy = 1$, find $\frac{du}{dx}$. 5M C05 L1

OR

- 11 a) Find the dimensions of the rectangular parallelepiped box open at the top of maximum capacity whose surface area is 108 square inches. 5M C05 L2
- b) Prove that $u = \sin^{-1} x + \sin^{-1} y$, $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ are functionally dependent and find the relation between them. 5M C05 L1

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CO: Course Outcome

BL - Blooms Taxonomy Levels